Inverse Calculation Method for Piezocomposite Materials Characterization.

G. Férin, Member, IEEE, D. Certon, J. Guyonvarch, and N. Félix.
VERMON, Tours - FRANCE; http://www.vermon.com
LUSI\ GIP Ultrasons, Tours - France

Abstract—The determination of elastic, dielectric and piezoelectric homogenised constants of piezocomposite materials is required for the optimisation and the performance evaluation of the material. This paper presents an inverse method enabling the determination of the effective electro-acoustic tensor [C, e, \(\varepsilon\)]. Data are displacement field measurements. The model used for the forward problem is presented. The calibration of the inversion process is done. A piezocomposite plate device is fully characterized. The determined parameters from the inverse problem are validated using other data, acoustic transmission coefficient of the plate.

Index Terms—Inverse calculation, piezocomposite materials.

I. INTRODUCTION

Piezocomposite materials are widely used for ultrasound imaging probes for medical or N.D.T. applications. All the components of the probe have importance on the final performances, but the main is the active layer. Piezocomposite structures offer many degrees of freedom that can be tuned as a function of the chosen performances. Characterization is a key issue for the piezocomposite optimization. The measurement of the composite material parameters ensures a better control of the manufacturing process and provides input data of any model (finite element or analytical approaches). This paper describes a method for determining the material parameters. This is an inverse problem based on the fit of experimental data. Considering a 1-3 piezocomposite material, containing a piezoelectric phase and a polymer phase (Fig. 1), the spatial averaging allows considering the material as a homogenized piezoelectric material with an equivalent symmetry class (6mm). Thus, 9 effective parameters have to be determined:

- 4 stiffness constants \((C_{11}^{\text{eff}}, C_{13}^{\text{eff}}, C_{33}^{\text{eff}}, C_{55}^{\text{eff}})\);
- 3 piezoelectric constants \((e_{31}^{\text{eff}}, e_{33}^{\text{eff}}, e_{15}^{\text{eff}})\);
- 2 dielectric constants \((\varepsilon_{11}^{\text{eff}}, \varepsilon_{33}^{\text{eff}})\).

Mechanical and dielectric losses are taken into account introducing complex mechanical stiffnesses and complex dielectric constants. The described approach uses displacement measurements, performed in front of a transducer, that are then fitted with a non linear least square solver available in the MATLAB Optimization Toolbox. The experimental setup and the forward problem are first presented. Then the different stages of the inversion calculation are described. Finally, the determined parameters are validated comparing simulation results with other experimental data, i.e. transmission coefficient through a piezocomposite plate for different incident angles of the acoustic beam.

II. EXPERIMENTAL SETUP

A. System Geometry.

![Device under test. One piezocomposite layer coated with a high reflective material.](image-url)

Fig. 1. Device under test. One piezocomposite layer coated with a high reflective material.

A piezocomposite plate, with 55% ceramic volumic fraction, having a 24mm*36mm rectangular section has been manufactured. The piezocomposite structure was designed for a curved abdominal imaging array. The thickness mode of the plate was 3.5 MHz. The microstructure pitch was chosen in order that the first lateral mode resonance be far enough from the bandwidth of interest, i.e. higher than 12 MHz. The piezocomposite layer had one electrode patterned on the upper face; the other was totally metallized and grounded. No matching layers and backing layers were used. A 6µm thick gold coated...
Mylar film was bonded on the array front face, i.e. grounded face, for increasing the light reflectivity of the surface (Fig. 1).

B. Displacement Measurement.

The displacement measurements have been performed using a compact Mach Zhender heterodyne interferometer equipped with a 100µm pumped diode and a doubled YAG laser (from Thales laser, France). The probe is optically focused to achieve a good lateral resolution. Absolute measurement of the mechanical displacement can be achieved in a frequency range from 20KHz to 18 MHz. The displacement of transducers is controlled through an IEEE-488 Interface using an OWIS DC-500 micro controller. Acquisitions were made using a digital oscilloscope, and then data were collected with a P.C. through an IEEE-488 interface. The electrical excitation was provided by a high bandwidth pulser [1].

The response of the plate submitted to such excitation condition must be calculated using the two stages described by Milson [4]. First, the electroacoustic Green function is computed considering than the charge density, on the face $z=+h/2$, is $\sigma(x,+h/2,t) = \delta(x) e^{i\omega t}$ where the $\delta$ function is the Dirac function and $\omega$ the working angular frequency. Next, the time pulsation term will be omitted for clarity. The potential at $z=+h/2$ and displacement field at $z=-h/2$, induced by the charge density excitation, are respectively noted $G_{\sigma\phi}(x,\omega)$ and $G_{\sigma\sigma}(x,\omega)$. These Green functions are easy to determine coupling the solutions of Christoffel equations to the electrical boundary conditions [5].

Then, for any charge density distribution $\sigma_0(x,\omega)$, The potential at $z=+h/2$ and displacement field at $z=-h/2$ writes respectively :

$$\Phi_{\sigma\phi}(x,\frac{h}{2},\omega) = \sum_{j=1}^{M} A_{ij} \sigma_{0j}(\omega), \quad i=1...M,$$

where $*$ indicates spatial convolution.

III. FORWARD PROBLEM

The model used to compute theoretical mechanical displacement has already been detailed [2] and [3] and will be briefly described here. The aim of the calculation is to compute all fields in the piezocomposite plate when submitted to a localized electrical source.

The plate vibrates in air, thus all mechanical stresses on the lower and the upper faces are null:

$$\begin{align*}
T_{33}(x,\pm\frac{h}{2},t) &= 0 \\
T_{31}(x,\pm\frac{h}{2},t) &= 0
\end{align*}$$

The metallized face is grounded, we have :

$$\phi(x,\frac{-h}{2},t) = 0$$

The electrical source is introduced thanks to the electrical boundary conditions expressed on the face where the single electrode is patterned. Electrical boundary conditions are non uniform since we have (Fig. 2) :

$$\begin{align*}
\{- \frac{L}{2} \leq x \leq + \frac{L}{2} \} & \quad \phi(x,\frac{h}{2},t) = v(t) \\
\{x > + \frac{L}{2} \} & \quad \sigma(x,t) = 0
\end{align*}$$

where $v(t)$ is the time voltage excitation, $\phi$ refered to the potential field and $\sigma$ to the charge density.

$$\begin{align*}
\sigma = 0 \quad & x = \pm\frac{L}{2} \\
\Phi = v(t) \quad & x = \pm\frac{L}{2} \\
\sigma = 0 \quad & z = \pm\frac{h}{2}
\end{align*}$$

Fig. 2. Detailed description of the electrical boundaries.

The plane wave decomposition of the displacement is easy to obtain thanks to a 2D Fourier transform of the x-t diagram. The spatial coordinate $x$ becomes wave vector $k$ and the time quantity becomes angular pulsation $\omega$, the new domain is called $\omega-k$ diagram. The interest of such diagram is the possibility to observe and to identify Lamb modes, propagating and evanescent modes.

where $v(t)$ is the time voltage excitation, $\phi$ refered to the potential field and $\sigma$ to the charge density.
A set of \( N \) sampling points are defined on the grounded face regularly spacing of \( \Delta x \). The equation (4.b) becomes:

\[
0 = \sum_{j=1}^{M} B_{ij} \sigma_{ij} (\omega), \quad i = 1 \ldots N,
\]

Eq. 6

where \( B_{ij} \) coefficients are equal to \( \Delta x G_{i\sigma} (x_i - x_j, \omega) \).

Finally, we have to compute the charge density \( \sigma(x, \omega) \) or the \( \sigma_{ij}(\omega) \) values required for the potential be constant along the electrode and equals to the voltage excitation, \( \phi_i(\omega) = V_0 \). Using equation 5, we write:

\[
\sigma_{ij}(\omega) = \sum_{j=1}^{M} A_{ij} \phi_j (\omega) = \sum_{j=1}^{M} A_{ij} V_0, \quad i = 1 \ldots M
\]

Eq. 7

where the \( A' \) matrix is the inverse matrix of \( A \).

The displacement field induced by the voltage excitation expresses as:

\[
u_i(\omega) = \sum_{j=1}^{M} B_{ij} \sigma_{ij} (\omega), \quad i = 1 \ldots N
\]

Eq. 8

From the calculated displacement field \( u_i(\omega) = u(x_i, \omega) \), the plane wave decomposition is obtained using a 1D Fourier transform along the \( x \) space domain, provided that the spatial sampling point number \( N \) be large enough.

### IV. INVERSE PROBLEM

The inverse problem aims to determine the elastic, piezoelectric and dielectric constants of the piezocomposite plate comparing the theoretical normal displacement field in the \((\omega, k)\) domain to the experimental one. This comparison is formulated as an optimization problem, which minimize the discrepancy between the computed and the measured normal displacement field. There are various optimization schemes that can be used to solve data fitting problem. We have used a non linear least square algorithm to determine material parameters.

#### A. Theoretical Inversion

Before leading an inverse problem from experimental displacement field, the stability and robustness of the inversion process must first be tested, on the base of theoretical displacement field. For this, a numerical phantom has been calculated from a given set of parameters. The fitting process has been performed from initial parameter values 40% higher than the “true”. The iterative process was successful after 2500 iterations. All the parameters have converged to their right value, with different convergence speed. Typically, the elastic constants requires 600 iterations while the piezoelectric and dielectric constants requires 2500 iterations (Fig. 4). The validity of the final values shows that solution can not be trapped in a local minimum point while the initial value set is near from 40% of the right values.

![Fig. 4. Variations of \( C_{33}^E \), \( e_{33} \), \( \varepsilon_{33}^S / \varepsilon_0 \) parameters as a function of the iteration index. The horizontal line represents the “right” value.](image)

#### B. Experimental inversion

The inverse problem has been conducted from the collected data. The initial values used were the effective parameters provided by the homogenization model of W.A. Smith [6]. The resin properties have been experimentally determined and the ceramic ones were taken from the manufacturer data sheet. The \( \omega-k \) diagram of the displacement obtained from the final values provided by the inverse problem is presented in figure 5. The measured \( \omega-k \) is presented on the same figure. On these diagrams, the \( S_0 \) Lamb mode is clearly observable. We obtain a good agreement between the minimized displacement diagram and the experimental one. The symmetric Lamb waves are very similar and the position and the width of the main lobe are well defined.
C. Experimental validation.

To validate the minimized set of parameters, an other experimental measurement has been performed. From a plate made with the same piezocomposite material, we have measured its transmission coefficient for several angles of the incident acoustic beam, as described in [7]. This is a well known method in the field of the mechanical characterization. Experimental results are shown in figure 6.b as a function of the angle and frequency.

Based on the model developed for the forward problem, the calculation of the transmission coefficient was easy to achieve. The theoretical transmission diagram obtained from the “found” parameters is presented in figure 6.a. On the two figures, the contributions of the Lamb waves are observable. We can notice that the same modes sawn in displacements are presents. Up to thirty degree angle, the two diagrams are similar. Showing that all the 33 parameters and the $C_{53}^E$ stiffness have been accurately obtained. The other quantities mainly affect weakly radiated modes which of course contribute to cross talking and spurious resonances. This explains why for large angle small discrepancies arise between the two figures.

V. CONCLUSION

An operational determination method of piezocomposite effective parameters has been described. We have tested the robustness of the inversion process using a numerical phantom. A 1-3 piezocomposite plate has been fully characterized. The determined values have been validated using another experimental measurement. Piezocomposite effective parameters will be implemented into a database which will be used for analytical or numerical optimization tools of probe.

REFERENCES