Abstract— A simple distributed model is presented and compared with FEM model. A good agreement is observed for collapse voltage and resonance frequency values. “Master curves” to helpful cMUT design such as normalized capacitance and coupling factor are defined as a function of normalized bias voltage. Next, the cMUT dynamic behavior is investigated through its electrical impedance and comparison is made with measurements.

Keywords-component: Micromachined capacitive transducers; finite differente model.

I. INTRODUCTION

Capacitive micromachined transducers [1] are now considered to show a high technological potential to make high performance air-coupled or water-coupled transducers. Precise modeling of their behavior have to be made during the design process prior to engineering and after for experience feedback. The influence, in terms of final performances, of both the geometrical parameters and mechanical properties of the c-MUT layers have to be known. Geometrical parameters include shape, size and thickness of the diaphragm, electrode and gap, mechanical properties mainly being flexural rigidity and residual stress linked to the manufacturing process. Different modeling approaches can be found in literature from the most simple one : 1D mass-spring analysis, up to large scale computing using finite elements models (FEM) for calculation of the electrical and the mechanical fields including the radiated acoustic field. Our objective has been to build up a calculation process in between these two extremes : accurate enough for being used in design engineering and avoiding massive electromechanical FEM

II. MODELING OF THE CMUT

The mobile part of the transducer is made with a silicon nitride membrane partially covered with an aluminum electrode, respectively noted (rm, hm) and (re, he) for the radius and thickness. The vertical displacement of the membrane is noted z(r) where r corresponds to the radial position. Each material is characterized by its mechanical and dielectric properties. E_m, v_m are respectively the Young modulus and the Poisson coefficient of the membrane. E_e, v_e are those of the electrode. ρ_m and ρ_e are the density of the two layers. ε_r is the effective permittivity of the membrane.

A. Mechanical Equation of the cMUT

When a tension V(t) is applied on the electrode, the membrane is submitted to an electrostatic pressure P_e(r,t). The movement of the membrane creates a radiated pressure P_r(r,t) in the medium surrounding the cMUT.

Assuming the membrane is uniform and neglecting the presence of the electrode, the plate equation governing the membrane vibration ([2], [3] and [4]) is given by Eq.1:

$$\frac{E_m \times h_m}{12(1-v_m^2)} \Delta z(r) + P_e(r,t) + P_r(r,t) = \rho_m h_m \frac{\partial^2 z(r)}{\partial t^2}$$  

where E_m, v_m, ρ_m and h_m are respectively the Young’s modulus, the Poisson’s ratio, the density and the thickness of the membrane material. The $\Delta$ operator is the Laplacian operator expressed in cylindrical coordinate. No damping term is introduced at this stage of the calculation, it will be done next. The parameter $D_m = \frac{E_m \times h_m}{12(1-v_m^2)}$ is the flexural rigidity of the plate. In our case, the flexural rigidity D(r) varies as a radius function. For a coordinate r outside the electrode, D(r) is equal to D_m. On the electrode D(r) is an homogenized flexural rigidity D_h which depends on the two layers characteristics and their thickness. The calculation of D_h can be calculated thanks to the popular relations used in the field of structural mechanic [3]. To take into account the variations of D(r) for the calculation of z(r), equation 1 is replaced by the equation of a non uniform plate, pre-stressed with a tensile stress T :

$$K_{plu} [z(r)] + P_e(r,t) + P_r(r,t) = \rho (r) h(r) \frac{\partial^2 z(r)}{\partial t^2}$$  

p(r) is the variation of the density with r and h(r) the variation of the total plate thickness. $K_{plu}$ is the derivative operator for modeling the elastic forces induced by the deformation of a non homogeneous plate. It expresses as :

$$K_{plu} [z(r)] = \Delta [D(r) \Delta(z(r))]$$  

$$= \frac{\partial (D(1-v))}{\partial r} \frac{\partial^2 z(r)}{\partial r^2} + \frac{\partial (D(1-v))}{\partial \theta} \frac{\partial^2 z(r)}{\partial \theta^2} + \frac{\partial^2 (D(1-v))}{\partial \theta^2} \frac{1}{r} \frac{\partial z(r)}{\partial \theta}$$
The cMUT membrane is assumed to be clamped on its perimeter, thus two boundary conditions are required:

\[
\begin{align*}
\frac{\partial z}{\partial r} & = 0 \\
\frac{\partial z}{\partial z_{\text{eq}}} & = 0
\end{align*}
\]  
(3)

To compute the electrostatic pressure \( P_e(r,t) \), we assume that the electrical field inside the cavity is mainly oriented along the vertical axis. Thus, \( P_e(r,t) \) writes [4]:

\[
P_e(r,t) = \frac{1}{2} \rho_c \left( \frac{V^2}{R_m + z(r,t)} \right) 
\]
(4)

where \( O(t) = \left\{ \begin{array}{ll} 1 & 0 \leq r \leq R_m, \ V \mbox{ is the excitation voltage and } 0 \mbox{ elsewhere} \\
C(r) & \mbox{ is the lineic capacity. We have : } C(r) = V \times \sigma(r) = \frac{\varepsilon_0}{\varepsilon_r + z(r)} \times O(t), \ \sigma(r) \mbox{ being the charge density along the membrane.}
\end{array} \right. \)

B. Solving with finite difference schema

This problem can not be solved analytically. We have chosen to use simple finite difference schema [5] to compute static and dynamic components of the charge density and the mechanical displacement. After differentiation the static and dynamic components of the charge density and the electrostatic pressure and the lineic capacitance. More, all

III. THEORETICAL RESULTS AND DISCUSSION

In this part, we want to compare simulation results provided by the finite difference model with those previously published such as FEM models by different authors.

A. Resonance frequency and collapse voltage

Figure 1.a shows, for cMUT configuration proposed by Caronti et al. [4], the variations of the collapse voltage as a function of the electrode radius normalized by the membrane radius. Theoretical values are in agreement with those calculated by Caronti. This curve shows that the simulated configuration presents low collapse voltage variations for normalized radius electrode ranging from 0.7 to 0.9.

Figure 1.b shows the variations of the mechanical resonance frequency as a function of the normalized electrode radius for different electrode types (Aluminium, Chromium and Copper). Each resonance frequency value has been normalized by the resonance frequency of the non metallized membrane. For this membrane configuration, the un-metallized membrane of one cMUT cell vibrating in air:

\[
Y(\omega) = j\omega C_0 + \frac{4 j \omega}{V_0} \left( \frac{\partial C}{\partial z} \right) \left[ K_{mm} - j \omega \sigma^2 \right]^{-1} \left[ \frac{\partial C}{\partial z} \right] (9)
\]
B. Static capacitance and Electromechanical coupling factor

In this section, several thickness of the membrane and gap height in the 200 to 400 nm range with the same metallization rate (50%) have been simulated. In order to compare theoretical results, a normalized capacitance taking into account the surface and the effective thickness ($h_{eq}$) of the cMUT membrane has been defined by

$$C_{Normalized}(V) = C_{0\text{-element}}(V) \times \frac{h_{eq}}{h_{membrane}}.$$  

The Figure 2.a reports the variation of the normalized static capacitance. We observe that all curves are super-imposed for a same metallization rate, showing that the static capacitance does not depend on the thickness of the different layers of the cMUT. At zero bias, the normalized capacitance is equal to $8.854 \times 10^{-12}$ F/m (equivalent to vacuum dielectric constant). In other respects, the influence of the membrane size with a given normalized electrode radius (50%) leads to the same results. The electromechanical coupling factor shows clearly the same curve for all configuration (Figure 2.b).

C. Dynamic behavior of the cMUT

The dynamic behavior of cMUT device is investigated through the electrical impedance of cMUT vibrating in air. Simulations are compared with experimental measurements obtained from cMUT devices which the membrane thickness was 410 µm, the gap height was 200 µm and the aluminum electrode was 200 nm thick. The radii of the electrode and the membrane were respectively 7 µm and 10 µm. The mechanical and electrical materials properties used in simulations are listed in Table I.

For impedance measurements a vector impedance analyzer (HP 4195 A) has been used. A plot of the measured and simulated electrical impedance of the transducer in air in the 5-30 MHz range is shown in Figure 3 for a 0.6 normalized bias voltage. The electromechanical coupling coefficient has been determined from the measurement of the $C_0$ capacitance value performed at frequency higher than the mechanical resonance. We have chosen 25 MHz. The comparison between theory end experiments is given in figure 4. The maximum $k_t^2$ value we have measured is 0.27. This small value is due to the presence of a high parasitic capacitance (630 % of the useful capacitance).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Silicon nitride</th>
<th>Aluminum</th>
</tr>
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<tbody>
<tr>
<td>Young’s modulus (GPa)</td>
<td>322</td>
<td>67.6</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.263</td>
<td>0.35</td>
</tr>
<tr>
<td>Density (kg.m$^{-3}$)</td>
<td>3270</td>
<td>2700</td>
</tr>
<tr>
<td>Relative permittivity</td>
<td>7.5</td>
<td></td>
</tr>
</tbody>
</table>

TABLE I. ELECTRICAL AND MECHANICAL PROPERTIES OF MATERIALS.
CONCLUSION

cMUT arrays can be sufficiently described by finite difference method. All parameters (collapse voltage, resonance frequency, static capacitance and coupling coefficient) have been estimated in air operation. This type of model seems suitable for a fast optimization of the cMUT electrical impedance.

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REFERENCES